Wave-particle duality - The Mach-Zehnder interferometer

M. Betz, I. de Lima and G. Mussatto

Additional Informations

This material contains additional informations on the themes listed below:

- *historical and technical details about the Mach-Zehnder interferometer;*
- *interpretations of Quantum Mechanics;*
- *theoretical description of monophotonic states and their production in the laboratory.*

I. Development and functionality of the Mach-Zehnder interferometer

A. History

The interferometer known as *Mach-Zehnder interferometer* was initially developed independently in two distinct laboratories. Ludwig Zehnder was a Swiss physicist who lived from 1854 to 1949 and was a professor at the University of Freiburg. The paper in which he introduced the interferometer is

> *Ein neuer Interferenzrefraktor* by L. Zehnder, Zeitschrift für Instrumentenkunde, vol. 11, p. 275, 1891.

Ernst Mach, a famous Austrian physicist and philosopher, lived from 1838 to 1916 and, at the time he was working in Prague, developed the same instrument in collaboration with his son Ludwig. That work was published by the latter in the same journal:

> *Über einen Interferenzrefraktor* by L. Mach, Zeitschrift für Instrumentenkunde, vol. 12, p. 89, 1892.

B. Description

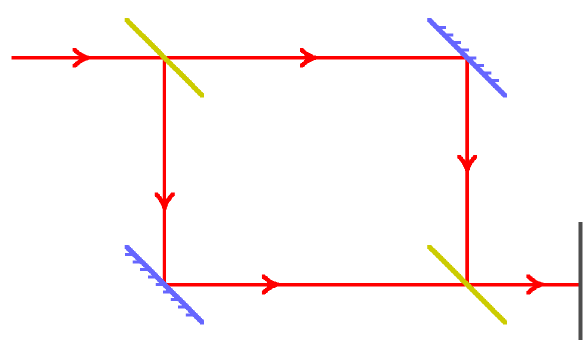

Figure 1: *Schematic representation of the Mach-Zehnder interferometer*

The interferometer is composed of two *semireflecting mirrors* and two totally-reflecting mirrors. The incident beam is divided in two components by the first semireflecting mirror. These components have equal intensity and propagate in perpendicular directions. After reflection by the reflecting mirrors, these components come together again at the second semireflecting mirror. The parallel arms of the instrument must be exactly equal. In the visualization, as well as in the discussion of the underlying mathematical formalism, it is assumed that the angle of incidence of a beam on a mirror is always $45°$ and that detectors are present at the apparatus exit. Actually, it is more usual to use a screen and to observe the interference pattern formed on it. For this to be possible, it is necessary that the incidence angle of the beams be slightly different from $45°$.

In the laboratory, the mirrors are usually set up vertically on a table. The beam propagates horizontally.

An excellent *Java simulation* of the interferometer was developed by the *University of Munich*. For a modified version (with interface in Portuguese), see here. That resource may be considered as a "realistic" simulation of the experimental device, in contrast to the "conceptual" visualization presented here.

C. Phase shifts due to reflections

The behavior of the interferometer is fundamentally determined by the *phase difference* between beam components when they interfere. Besides being possibly affected by the presence of a transparent material on the path of one of the beam components, this difference depends on the *phase shifts introduced by reflections*.

The phase-shift induced by a reflecting mirror depends on the type of material used to produce the reflection. In some mirrors, the reflecting layer is made of a dielectric material. In that case, if the refraction index of this layer is larger than the index of the medium in which the beam propagates, the reflection introduces a phase shift of π radians. In contrast, if the layer's refraction index is smaller than that of the medium, no phase shift is introduced. There also exists mirrors whose reflecting layer is metallic and therefore conducting. In that case, the phase shift depends on the metal employed.

In the Mach-Zehnder interferometer, each beam component necessarily undergoes one, and only one, reflection by a totally reflecting mirror. For this reason, the phase shift introduced by such a reflection turns out to be in fact *irrelevant*. For *mere convenience*, it will be assumed, in the development of the formalism and in the commentaries accompanying the animations, that this phase shift has the same value as the phase shift associated to reflection by a semireflecting mirror, for which the value $\pi/2$ shall be adopted, for reasons discussed below.

In the case of a semireflecting mirror, the phase shifts introduced in the reflected and transmitted components also depend on the construction details of the mirror. The reading of the brief article

> *"How does a Mach-Zehnder interferometer work?"*, by K. P. Zetie, S. F. Adams and R. M. Tocknell, Physics Education 35, p. 46 (2000),

may be recommended. Besides a critical discussion, these authors present a calculation of the phase difference between the two beam components based on the following model of a semireflecting mirror: a blade made of a transparent material with one of its faces covered by a dielectric material of refraction index intermediate between that of the transparent material and that of air. They conclude that completely constructive interference will occur at the apparatus exit in the direction of the screen (see the figure above), whereas it will be completely destructive in the perpendicular direction.

A semireflecting mirror may be considered as a particular case of *beam splitter*. A general theoretical discussion of the phase shifts introduced by such devices, based only of the assumption of absence of attenuation, that is, on the conservation of the total radiation flux, is presented in the paper

> *"General properties of lossless beam splitters in interferometry"*, by A. Zeilinger, American Journal of Physics 49, p. 882 (1981).

This author shows that, if one denotes by δ_L the phase difference between the reflected and transmitted components for a wave that reaches the splitter coming from the left, and by δ_R the analogous quantity for a wave coming from the right, the relationship

$$\delta_L + \delta_R = \pi$$

holds. In the case of the mirrors considered by the authors previously cited, assuming that the dielectric material is on the left face, one has

$$\delta_L = \pi - \delta_T \ \text{ and } \ \delta_R = \delta_T \ ,$$

where δ_T is the phase shift produced by the crossing (at $45°$) of the blade of transparent material. This phase shift depends on the blade thickness and on the refraction index of the material.

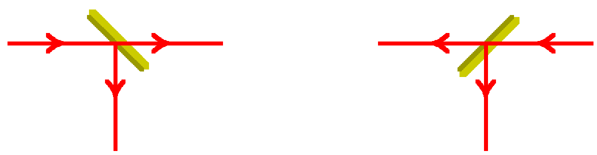

Figure 2: *Asymmetric semireflecting mirror.*

Obviously, such a semireflecting mirror is asymmetric in general. A *symmetric* mirror would be such that

$$\delta_L = \delta_R = \pi/2 \ ,$$

which may be obtained by choosing the blade's thickness such that $\delta_T = \pi/2$. The mathematical formalism accompanying the present material is based on the latter assumption.

It should be stressed that, in the Mach-Zehnder interferometer, the number of reflections by semireflecting mirrors will depend on the path followed, possible values being zero, one, or two, as can be easily verified. Therefore, it is essential to take into account the phase shifts due to reflections by these mirrors. However, the final result obtained with our assumption of symmetric mirrors is identical to that reached by the above cited authors on the basis of asymmetric mirrors.

II. Interpretations of quantum mechanics

A. Role of an interpretation

An *interpretation of a physics theory* may be defined as a set of rules relating its formalism to observed phenomena. Already at the beginning of quantum physics, it was noticed that the relevant phenomena - clicks in detectors, scintillations on a screen, etc - were not individually predictable. It was only possible to make predictions about statistical distributions of the results. Since, on the other hand, a wave function is a quite abstract mathematical object, not readily associated to the propagation of a directly observable physical quantity, it is not surprising that an interpretation emerged that relates the wave function to the probability of observing the associated particle. It was Max Born who first stated the precise rule, equaling the square of the modulus of the wave function to the probability of finding, at a given instant, the particle in a detector of unit volume localized at a given point.

The evolution of the wave function, described by the *Schrödinger equation*, is deterministic in the following sense: knowing the wave function at a given instant, it is possible to compute it at a later instant, *as long as nothing disturbs the system in the interval*. But a measurement in general introduces such a perturbation; what then happens is the most difficult question in the interpretation of quantum mechanics. It is frequently referred to as the *Measurement Problem* and it is mostly in discussions of this problem that controversies arise between various interpretations. The most well-known of these shall be briefly discussed below.

As a general reference, one may recommend the compilation

> *"Quantum Theory and Measurement"*, edited by J. A. Wheeler and W. H. Zurek, Princeton University Press, Princeton, USA (1983),

which shall be referred to using the shorthand QTM. There, are reproduced and commented the most significant works published on the subject, from the paper by Born already cited and the debate between Niels Bohr and Albert Einstein during the 1930's, until the advances achieved in the 1970's. It should be emphasized, however, that important progress has occurred since the publication of this compendium.

The Mach-Zehnder interferometer is used as illustration to discuss interpretations of quantum mechanics in the book

> *"Conceitos de Física Quântica"*, by O. Pessoa Jr., Editora Livraria da Física, São Paulo, Brasil (2003),

and in the paper

> *"Interpretações da mecânica quântica em um interferômetro virtual de Mach-Zehnder"*, by F. Ostermann and S. D. Prado, Revista Brasileira de Ensino de Física 27, p. 193 (2005),

where the discussion is based on the simulation of the apparatus mentioned above. [Both latter references are written in Portuguese.]

B. The Copenhagen - Von Neumann interpretation

Probably because of the preponderant influence of the Danish physicist Niels Bohr, the most commonly used interpretation, in teaching as well as in the practice of research, has become known as the *Copenhagen interpretation*. Frequently associated to it also is the name of the American mathematician John Von Neumann, who is responsible for its most systematic formulation. It should however be emphasized that numerous important physicists contributed to the development of this interpretation, often differing in the details of it.

In this interpretation, performing a measurement on a quantum system results in an abrupt modification of the state of the system, in such a manner that the state of the system after the measurement depends on the result which it yielded. In the case of a particle, this process is the so-called *wave-packet collapse*, such that after a measurement of the position of the particle, the packet's extension becomes limited to the region in which the particle was observed. Other parts of the packet simple "disappear". *It is this interpretation that is used in the computer visualizations of the present resource.*

It should be mentioned that the effect of a measurement on a system can, in principle, be studied in more details if the detector itself is considered as a quantum system interacting with the system under study (a particle, for example). It is easily shown that the interaction between the system and the apparatus will result in the final state being a coherent superposition of products of states of the system correlated with states of the apparatus. By such an approach, and taking into account the fact that a detector is generally a "macroscopic" system made up of many particles, it is possible to separate two aspects that, together, constitute the collapse:

i. The washing out of interferences between components of the final-state superposition corresponding to macroscopically distinct states of the apparatus. This is a physical phenomenon, usually dubbed *decoherence*.

ii. The selection of one of the components and the wiping out of all the others, on the basis of the result obtained in the measurement. The word *objectivation* is used in reference to this step, which must be understood, not as a physical phenomenon proper, but rather as a way of inserting in the formalism the information obtained in the measurement.

A detailed exposition of the theory of observation in quantum mechanics is presented in the classic paper

> *"The Theory of Observation in Quantum Mechanics"*, by F. London and E. Bauer, reproduced in the QTM collectanea, p. 217.

On decoherence and its importance for the emergence, in quantum mechanics, of the classical behavior of a system, one may consult

> *"Decoherence and the Appearance of a Classical World in Quantum Theory"*, by E. Joos, H. D. Zee, C. Kiefer, D. Giulini, J. Kupsch and I.-O. Stamatescu, Springer, Berlin (2003).

C. The many-worlds interpretation

The main objection to the Copenhagen interpretation is that it separates the quantum object under study from the apparatus employed or, at the minimum, from the observer who conducts the investigation. Such a separation is impossible in at least one case, namely when the object is the whole universe. It is therefore not surprising that cosmologists have been particularly inclined to seek alternatives.

The *many-worlds*, or *multiverse interpretation*, also known as the *relative-state interpretation*, associates a wave function to the whole universe and dispenses with the collapse altogether. According to it, what happens in a measurement is merely a *ramification* of the total universe into several components. Each component is a sub-universe in which the measurement result has a definite value and the observer is aware of that value only. Thus, we would be cohabiting, without being able to perceive it, with numerous alternative versions of ourselves and everything else.

On this interpretation, one may consult the review article written by its proponent,

> *"Relative State Formulation of Quantum Mechanics"*, by H. Everett III, Review of Modern Physics 29, 454 (1957),

which is reproduced in the already-cited [QTM colletanea, p. 315]. In the book

> *"The Many-Worlds Interpretation of Quantum Mechanics"*, edited by B. S. DeWitt and N. Graham, Princeton University Press, Princeton, USA (1973),

is reproduced the doctor thesis of Hugh Everett III, in which he put forward and developed the interpretation in question.

D. The consistent-histories interpretation

This is the youngest of interpretations. It was originally proposed in the article

> *"Consistent histories and the interpretation of quantum mechanics"*, by R. B. Griffiths, Journal of Statistical Physics 36, p. 219 (1984),

and has drawn growing interest. Histories are *successions of events* happening in a given system. They may be classified in families such that histories belonging to the same family satisfy *consistency conditions* that allow the application of the usual rules about conditional probabilities. Histories belonging to distinct families cannot be invoked in the same reasoning without falling into contradiction. Such a restriction is reminiscent of the notion of *complementarity* already advocated by Niels Bohr.

As the previous one, this interpretation has attracted the attention of cosmologists, as it permits to deal with a closed system, without the need of a separate apparatus or observer. It also has the virtue of clarifying the conditions in which the Copenhagen interpretation, which assumes such a separation, is applicable.

For more informations, one may recommend the didactic book written by the inventor himself,

> *"Consistent Quantum Theory"*, by R. B. Griffiths, Cambridge University Press, Cambridge, UK (2002),

as well as the text

> *"Understanding Quantum Mechanics"*, by R. Omnès, Princeton University Press, Princeton USA (1999).

E. The hidden-variables interpretation

It is well known that Einstein himself suspected that quantum mechanics might only be an incomplete description of a subjacent reality. One should then seek a more complete theory that would involve so-called *hidden variables*.

The question of the compatibility of a theory of this type with experimental facts has been discussed by several authors, in particular John Bell, who established *inequalities between probabilities* that should necessarily be satisfied by a *local* hidden-variables theory, but are not satisfied in quantum mechanics. In the 1980's, the French physicist Alain Aspect and his group performed experiments that demonstrated these inequalities are *violated by nature*, confirming that the latter conforms to the rules of quantum theory.

The possibility of interpreting quantum mechanics as a *non-local* hidden variable theory had already been pointed out by some authors, in particular David Bohm and also Louis de Broglie. In Bohm's theory, the fundamental evolution equation of quantum mechanics (the Schrödinger equation) is re-written in the form of Newton's second law for particles submitted to two different forces: the Newtonian force, derived from the usual classical potential, and an additional "force", derived from a *quantum potential* constructed in terms of the quantum wave function. The classical potential is normally a smooth function that vanishes outside the interaction range. In contrast, the quantum potential is in general a wildly oscillating function and is *not zero* in regions where the particles are free. It is these characteristics that account for the unpredictability of particle motion and the emergence of interference patterns in their distributions. In this interpretation, the evolution of the wave function influences the motion of the particles, but not the contrary. For this reason, the detection of a particle does not result in the collapse of the wave function.

The original paper

> *"A Suggested Interpretation of the Quantum Theory in Terms of Hidden Variables, I and II"*, by D. Bohm, Physical Review 85, p. 166 (1952)

is reproduced in the [QTM colletanea, p. 369]. An accessible presentation, accompanied by a critical assessment of interpretations of quantum mechanics from a historical and social point of view may be found in

> *"Quantum Mechanics: Historical Contingency and the Copenhagen Hegemony"*, by J. T. Cushing, The University of Chicago Press, Chicago USA (1994).

III. Photons

A. Quantization of the electromagnetic field

Corpuscular theories of light have been defended by several great physicists - including Newton - until, at the beginning of the XIX century, the observation of diffraction effects and interference impelled the adoption of an undulatory picture. The demonstration that Maxwell's theory implied the existence of waves propagating in vacuum with the expected speed, and the production of such waves by Hertz, completed the unification of optics and electromagnetic theory.

In 1900 however, in an attempt to explain *black-body radiation*, Max Planck was led to introduce the postulate of *energy quantization*. In his "annus mirabilis" of 1905, Einstein provided a basis for this conjecture, in a sense reviving the corpuscular vision of light. For Einstein, any electromagnetic radiation was made up of corpuscles that became known as *photons*. As is well known, based on this hypothesis, Einstein formulated a simple theory of the *photoelectric effect*, which was confirmed in the second decade of the XX century by experiments conducted by Millikan.

As this historical summary shows, *wave-particle duality* was established for radiation before being postulated and verified for matter. The latter took place in the third decade of the XX century, with the founding theoretical work of de Broglie and the observations of Davisson and Germer. From that moment on, quantum theory was developed quite rapidly.

In the present view of fundamental physics, all processes, those involving material particles - electrons, quarks, etc - as well as those involving radiation - photons - are described by *quantum field theory*. Ordinary quantum mechanics is merely an approximation valid for phenomena involving material particles at low energy. The expression *first quantization* is commonly used in reference to this approximation and the expression *second quantization* for the more general theory. It is a curious fact that the basic idea of second quantization - the photon hypothesis - was introduced before the starting point of first quantization - the postulate of the existence of matter waves. The consequences of this for didactics may be perplexing: a professor of quantum mechanics at the introductory level will state with conviction that "only with the photon hypothesis is it possible to explain the laws of the photoelectric effect". In contrast, a professor - possibly the same person - teaching a more advanced course might well choose the photoelectric effect as an illustration of time-dependent perturbation theory. He will then be able to derive the correct laws by merely quantizing the electron position, but treating the electromagnetic field as classical.

For a critical discussion of the experimental evidence for the quantization of radiation, see

> *"The Quantum Challenge: Modern Research on the Foundations of Quantum Mechanics"*, by G. Greenstein and A. G. Zajonc, Jones and Bartlett Publishers, Sudbury USA (1997).

B. A photon wave function?

After invoking the concept of the photon to analyze in a simple manner a few interaction processes of radiation with matter, such as the photoelectric effect and the Compton effect, a first course on quantum mechanics typically turns to the quantization of matter, starting with a detailed discussion of the Schrödinger equation. Photons mostly leave the scene, until an advanced graduate level is reached, when the systematic development of quantum electrodynamics can be undertaken.

However, it is generally assumed that the basic concepts of quantum physics – principle of superposition, principle of indeterminacy, measurement process – may be discussed indifferently with photons, electrons, neutrons, etc. If one chooses, as in the present contribution, to consider light and photons, one is faced with the necessity to describe them at the level of "first quantization". In particular, if one wishes to analyze what happens when photons go through the interferometer "one at a time", one needs to be able to associate a wave packet to an individual photon.

A survey of the literature reveals that the attribution of a wave function to a photon has been considered by several authors, under rather diverse angles. In the classic paper

> *"Localized States for Elementary Systems"*, by T. D. Newton and E. P. Wigner, Reviews of Modern Physics 21, p. 400 (1949),

conditions are formulated that are required for it to be possible to attribute a definite position to a particle in relativistic quantum mechanics. It is shown that these conditions cannot be fulfilled in the case of massless particles of unit spin, such as the photon. On the basis of this work, the possibility of associating to the photon a wave function and a position probability distribution has been considered dubious, at best.

Despite this, some authors have tried to ground on quantum electrodynamics theoretical constructions of probability distributions for photons. In the paper

> *"Photon Dynamics"*, by R. J. Cook, Phys. Rev. A, 25, p. 2164 (1982),

a formulation in terms of two vector fields, dubbed *photon fields*, is derived. From it, is deduced a quantity possessing the interpretation of "granular" density, that is, specifying the localization probability of a photon, but only in regions of size much larger than the wave length and in time intervals much larger than the period. This theory has been recast in a form similar to usual quantum mechanics, introducing a six-components wave function, in the paper

> *"Quantum Mechanical Approach to a Free Photon "*, por T. Inagaki, Phys. Rev. A, 49, p. 2839 (1994).

An alternative approach consists in performing a "first quantization" of classical electrodynamics. Riemann had already proposed a formulation of the latter theory in terms of a complex field, whose real and imaginary parts were associated to the electric and magnetic fields, respectively. The interpretation of this complex field as a wave function is quite natural and is developed in the article

> *"The Photon Wave Function"*, by T. Bialynicki-Birula, in Coherence and Quantum Optics VII, ed. by J. H. Eberly, L. Mandel and E. Wolf, Plenum, New York, USA (1996).

The statistical density thus deduced should be interpreted as providing the probability of measuring energy at a given place.

All in all, and despite some limitations and difficulties of interpretation, it seems legitimate to associate a wave function to a photon. In the other part of this supporting material, in which the relevant mathematical formalism is developed, a simplified wave function is employed, which describes a wave packet propagating with the speed of light and assumed to admit the Born probabilistic interpretation.

C. Monophotonic states

In the visualization and discussion of the quantum corpuscular aspects, the passage of each individual photon through the interferometer is traced. This assumes that it is possible to localize with adequate precision, in space and in time, a single photon. Although the emission of a single photon by an excited atom is a common process, such a photon is not localized. The production of the state of a single localized photon requires an experimental device specifically designed for this purpose. The procedure proposed – and successfully tested – in the article

> *"Experimental Realization of a Localized One-Photon State"*, by C. K. Hong and L. Mandel, Phys. Rev. Lett. 56, p. 58 (1986).

shall be discussed briefly.

The physical phenomenon employed, known as *spontaneous parametric down conversion*, occurs when a coherent radiation beam penetrates a crystal which is not symmetric under inversion. In that process, a photon of the incident beam is "divided" in two photons of larger wave length. The sum of the energies of these two photons is equal to the energy of the initial photon, and they are produced essentially simultaneously (within an interval inferior to $100\,ps$). The propagation directions of the produced photons are different (and also different from the direction of the incident beam), but correlated. As an example of a crystal producing this effect, one may cite potassium dihydrogen-phosphate (KH_2PO_4, known as *KDP*).

In order to be able to claim that a photon is traveling through the interferometer, and to say where it is located at a given instant, it suffices to insert a crystal with these properties on the beam path and detect one of the produced photons with a control detector D_C. It will then be possible to deduce where the other photon – which entered the interferometer – is to be found.

With adequate electronics, it will be possible to utilize D_C to habilitate the other detectors (for example detector D_1 set at the exit, and/or detector D_3 placed in one of the arms, see the figure below) only at the right moment to observe the desired photon.

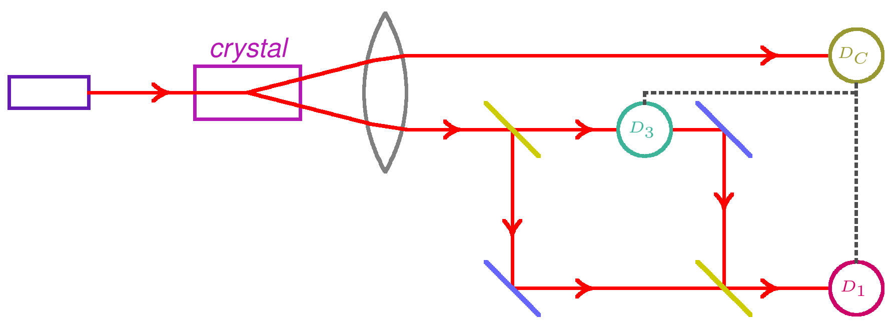

Figure 3: *Utilization of down parametric conversion to monitor the passage of a photon.*